A Discrete Model for Inelastic Deformation of Thin Shells

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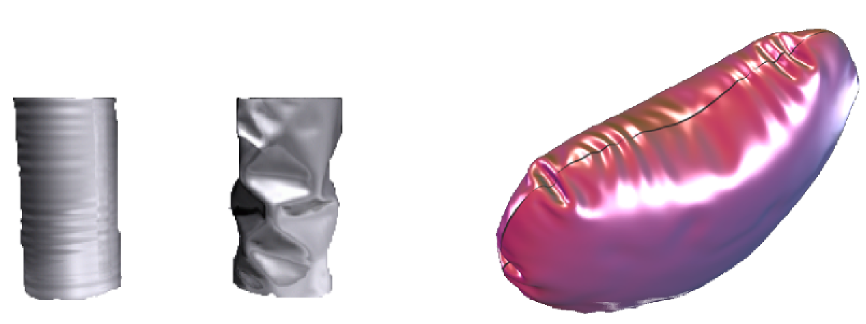
Thin shells

Thin shells: thin, curved, flexible surfaces

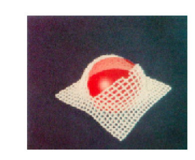
Good approximation for real-world objects

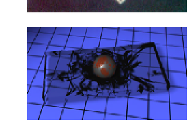
Applications in entertainment, medicine, etc.



Context

Terzopoulos and Fleischer 1988
- Thin plates using splines
- Fracture followed mesh edges





O'Brien and Hodgins 1999, 2002
- Volumetric meshes for brittle and ductile fracture

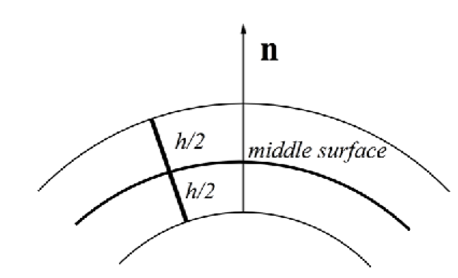
Grinspun, Hirani, Desbrun and Schröder 2003
- Discrete models using invariants

Cohen-Steiner and Morvan 2003
- Discrete curvatures

Motivation

Simulation of thin shells
- Simple to implement
- Derivation from fundamental elasticity theory
- Elegant discretization
- Captures wide range of materials

Simple shell model
- Negligible deformation in the normal direction



Contributions

New discrete bending strain
- Expressed in terms of mesh invariants

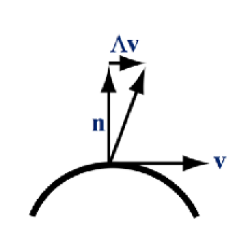
Applications
- Elasticity
- Plastic flow
- Fracture

Algorithmic enhancements
- Search for fracture and collision events
- Vertex budging
- Collision response with fracture

Shape Operator

2nd order tensor — Λ

For any tangent vector v on the surface, Λv is the derivative of the surface unit normal in the direction of v



Can be diagonalized for
- The principal directions of curvature
- The principal magnitudes of curvature

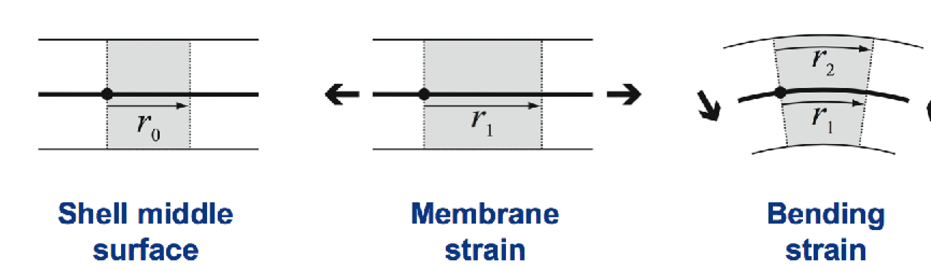
Membrane and bending strains

Deformations of the shell middle surface

Integrated over the shell thickness

Membrane strain: in-plane stretching

Bending strain: out-of-plane deformation



Membrane and bending energy

Membrane energy:

$$W_m = \frac{Yh}{2(1-\upsilon^2)}\left((1-\upsilon)\mathrm{Tr}(E_m^2) + \upsilon(\mathrm{Tr}\,E_m)^2\right)$$

Bending energy:

$$W_c = \frac{Yh^3}{24(1-\upsilon^2)}\left((1-\upsilon)\mathrm{Tr}(E_c^2) + \upsilon(\mathrm{Tr}\,E_c)^2\right)$$

Both energies are of the same form
- Direct consequence of novel bending strain
- Membrane and bending scale by h and h^3, respectively

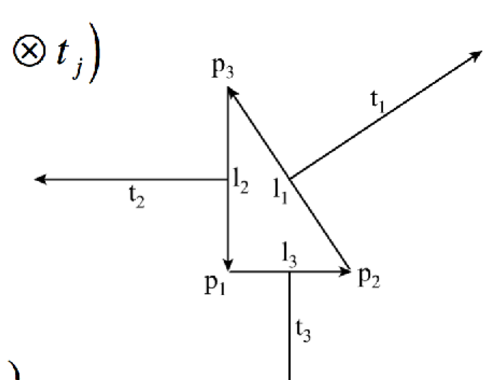
Discrete strains

Discrete membrane strain
- Measures change in squared length

$$E_m = \frac{1}{8A^2}\sum_i \left(\tilde{l}_i^2 - l_i^2\right)\left(t_j \otimes t_k + t_k \otimes t_j\right)$$

Discrete bending strain
- Measures change in curvature magnitude *and* direction

$$E_c = \tilde{\Lambda} - \Lambda = \frac{1}{2A}\sum_i \frac{\varphi(\tilde{\theta}_i) - \varphi(\theta_i)}{l_i}\left(t_i \otimes t_i\right)$$

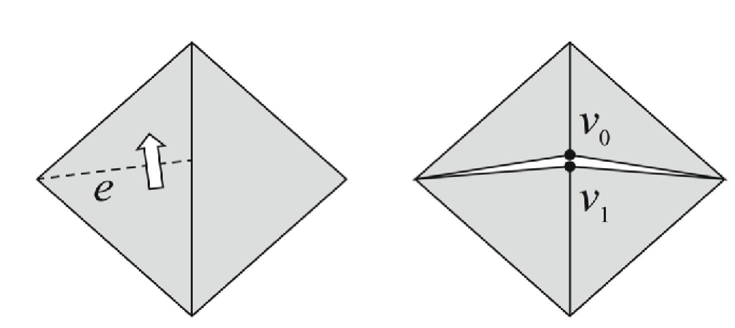


Fracture

The surface fails when the strain exceeds threshold

The fracture direction is perpendicular to the largest eigenvalue of the strain

Insert a split in the triangle and its neighbor

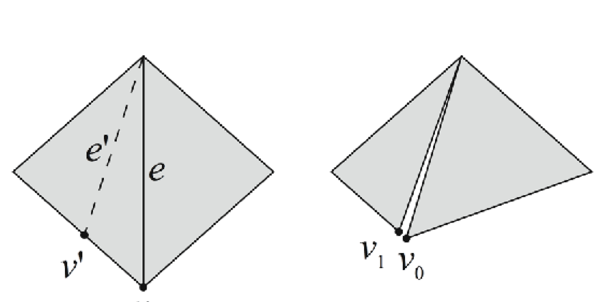


Budging

Fracturing near existing edges can introduce sliver triangles

Vertex budging reparameterizes the mesh

Move the vertex v to a location v' on the fracture line



Results: Light bulb

Glass light bulb anchored into a rigid base

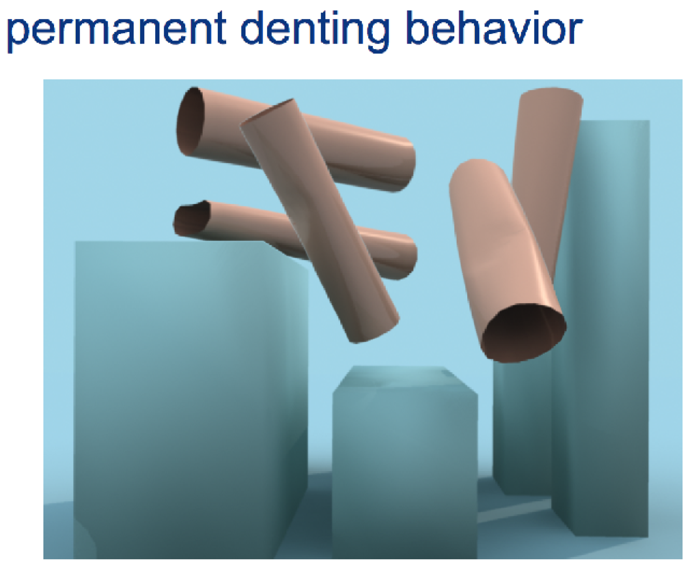
Struck by a rigid metal ball

Fracture, collisions and dynamics

Results: Plasticity

Metal tube with plastic absorption of energy

Realistic permanent denting behavior



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